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Hidden LocaL Symmetry Theory as an Effective Field Theory of QCD *

Masayasu Harada

Department of Physics, Nagoya University, Nagoya 464-8602, Japan

Abstract

In this write-up, I summarize the recent development of the hidden local symmetry (HLS) theory as an effective field theory of QCD. I first explain how the systematic chiral perturbation in the HLS is justified in the large N_c limit of QCD, and summarize the basic concept of the Wilsonian matching, through which some of the bare parameters of the HLS are determined by matching the HLS to the operator product expansion in QCD. Then, I briefly review how to formulate the vector manifestation in hot matter.

1 Introduction

Quantum Chromodynamics (QCD) is known to be a fundamental theory for describing the low-energy hadron phenomena. However, it is very difficult to reproduce experimental data directly from QCD, since QCD is the strong coupling gauge theory. Then, instead of studying QCD directly, it is convenient to use effective field theories (EFTs) written in terms of hadronic degrees of freedom. When one studies the phenomena of light hadrons, it is important for EFTs to reproduce the chiral symmetry properties of QCD: The Lagrangian of QCD in the light quark sector possesses the approximate chiral symmetry which is spontaneously broken down to the flavor symmetry. As a result, the pion appears as the approximate massless Nambu-Goldstone boson. Furthermore, there must exist a systematic expansion scheme in an EFT, by which one can systematically include higher derivative terms together with loop corrections.

One of popular EFTs is the chiral perturbation theory (ChPT) [1, 2], in which the pion is the only relevant degree of freedom. Starting from the energy region around the pion mass, one can study the higher energy region systematically by including the higher derivative terms together with the loop corrections. In much higher region, however, we know the existence of ρ meson and the ChPT may not be applicable in the energy scale beyond the ρ meson mass. One simple way to study the hadron phenomena in such an energy region is to include ρ meson in addition to the pion and make an EFT.

There are several ways to include the rho meson into effective Lagrangians in the literature: the matter field method [3]; the massive Yang-Mills field method [4]; the anti-symmetric tensor

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field method [2, 5]; and the hidden local symmetry (HLS) [6]. In the HLS, as first pointed in Ref. [7] and developed further in Refs. [8, 9, 10, 11], thanks to the gauge symmetry, a systematic loop expansion can be performed with the ρ meson included in addition to the pion. Furthermore, in Ref. [10], the Wilsonian matching was proposed as a way to determine some of the parameters by matching the HLS to QCD, which together with the ChPT with HLS was done in remarkable agreement with the experiments [10, 11, 12].

In Ref. [13], using the Wilsonian matching, we studied the chiral symmetry restoration in large flavor QCD [14], and proposed a new pattern of the chiral symmetry restoration, which we called the vector manifestation (VM). The VM is a novel manifestation of Wigner realization of chiral symmetry where the ρ meson becomes massless degenerate with the pion at the chiral phase transition point. In Refs. [15, 16], the Wilsonian matching together with the HLS was applied for the chiral symmetry restoration in hot and/or dense QCD [17, 18, 19], and the formulations of the VM in hot matter [15] and dense matter [16] were presented. The VM in hot and/or dense matter gives a theoretical support of the dropping mass of the ρ meson following the Brown-Rho scaling proposed in Ref. [20], which can explain (see, e.g., Refs. [21, 18, 19]) the enhancement of dielectron (e^+e^-) mass spectra below the ρ/ω resonance observed at the CERN Super Proton Synchrotron (SPS) [22].

In this write-up, I first summarize the recent development of the HLS theory as an EFT of QCD including the Wilsonian matching. Then, I will briefly show how to formulate the VM in hot matter.

This write-up is organized as follows: In section 2, starting from the Lagrangian of the HLS, I will briefly explain how the ChPT with HLS is justified in the large N_c limit of QCD. Next, I will introduce several essential ingredients of the Wilsonian matching in section 3. In section 4, I will briefly review the difference between the VM and the conventional manifestation of chiral symmetry restoration based on the linear sigma model. Section 5 is devoted to show how to formulate the VM in hot matter. Finally, in section 6, I will give a brief summary.

2 Hidden Local Symmetry

In this section I will briefly explain the hidden local symmetry (HLS) theory.

The HLS theory is based on the $G_{\rm global} \times H_{\rm local}$ symmetry, where $G = SU(N_f)_{\rm L} \times SU(N_f)_{\rm R}$ is the chiral symmetry and $H = SU(N_f)_V$ is the HLS. #1 The basic quantities are the HLS gauge boson and two matrix valued variables $\xi_{\rm L}(x)$ and $\xi_{\rm R}(x)$ which transform as

$$\xi_{\rm L,R}(x) \to \xi'_{\rm L,R}(x) = h(x)\xi_{\rm L,R}(x) g^\dagger_{\rm L,R} \ , \tag{1}$$

where $h(x) \in H_{\rm local}$ and $g_{\rm L,R} \in [{\rm SU}(N_f)_{\rm L,R}]_{\rm global}$. These variables are parameterized as

$$\xi_{\rm L,R}(x) = e^{i\sigma(x)/F_\sigma} e^{\mp i\pi(x)/F_\pi} \ , \tag{2}$$

#1 In this write-up, I consider the QCD with general number of flavors, i.e., I include N_f massless quarks. Nevertheless, I call the vector meson (ρ meson and its flavor partner) the ρ, and the pseudoscalar meson (pion and its flavor partner) the π.

where $\pi = \pi^a T_a$ denotes the pseudoscalar Nambu-Goldstone (NG) bosons associated with the spontaneous symmetry breaking of $G_{\rm global}$ chiral symmetry, and $\sigma = \sigma^a T_a$ denotes the NG bosons associated with the spontaneous breaking of $H_{\rm local}$. This σ is absorbed into the HLS gauge boson through the Higgs mechanism. F_π and F_σ are the decay constants of the associated particles. The phenomenologically important parameter a is defined as

$$a = \frac{{F_\sigma}^2}{{F_\pi}^2} \ . \tag{3}$$

The covariant derivatives of $\xi_{\rm L,R}$ are given by

$$\begin{aligned} D_\mu \xi_{\rm L} &= \partial_\mu \xi_{\rm L} - i V_\mu \xi_{\rm L} + i \xi_{\rm L} \mathcal{L}_\mu \ , \\ D_\mu \xi_{\rm R} &= \partial_\mu \xi_{\rm R} - i V_\mu \xi_{\rm R} + i \xi_{\rm R} \mathcal{R}_\mu \ , \end{aligned} \tag{4}$$

where V_μ is the gauge field of $H_{\rm local}$, and $\mathcal{L}_\mu$ and $\mathcal{R}_\mu$ are the external gauge fields introduced by gauging the $G_{\rm global}$ symmetry.

The HLS Lagrangian with the lowest derivative terms in the chiral limit is given by [6]

$$\mathcal{L}_{(2)} = {F_\pi}^2 \mathrm{tr}[\hat{\alpha}_{\perp\mu} \hat{\alpha}_\perp^\mu] + {F_\sigma}^2 \mathrm{tr}[\hat{\alpha}_{\parallel\mu} \hat{\alpha}_\parallel^\mu] - \frac{1}{2g^2} \mathrm{tr}[V_{\mu\nu} V^{\mu\nu}] \ , \tag{5}$$

where g is the HLS gauge coupling, $V_{\mu\nu}$ is the field strength of V_μ and

$$\begin{aligned} \hat{\alpha}_\perp^\mu &= \frac{1}{2i} [D^\mu \xi_{\rm R} \cdot \xi_{\rm R}^\dagger - D^\mu \xi_{\rm L} \cdot \xi_{\rm L}^\dagger] \ , \\ \hat{\alpha}_\parallel^\mu &= \frac{1}{2i} [D^\mu \xi_{\rm R} \cdot \xi_{\rm R}^\dagger + D^\mu \xi_{\rm L} \cdot \xi_{\rm L}^\dagger] \ . \end{aligned} \tag{6}$$

In the HLS, as first pointed in Ref. [7] and developed further in Refs. [8, 9, 10, 11], thanks to the gauge symmetry, a systematic loop expansion can be performed with the vector mesons included in addition to the pseudoscalar mesons. In this chiral perturbation theory (ChPT) with HLS one can show that the loop expansion corresponds to the derivative expansion as in the ordinary ChPT. Here, I show the expansion parameter and the order counting of the systematic expansion in the HLS. As is well known, the expansion parameters of the ordinary ChPT in the chiral limit is p^2/Λ_χ^2, where p is the typical momentum scale and $\Lambda_\chi \sim 4\pi F_\pi$ the chiral symmetry breaking scale. In addition to these two parameters, m_ρ^2/Λ_χ^2 is also the expansion parameter in the HLS. For the validity of the expansion in the parameter m_ρ^2/Λ_χ^2, we need to show that this expansion parameter is actually small, and that the quantum correction proportional to $1/m_\rho$ never appears.

Let me first consider the smallness of the expansion parameter m_ρ^2/Λ_χ^2. The smallness is actually justified in the large N_c QCD. As is well known, in the large N_c limit of QCD, the π decay constant scales as $F_\pi \sim \sqrt{N_c}$, while the ρ mass does not scale. So the ratio $m_\rho^2/(4\pi F_\pi)^2$ scales as $1/N_c$, and becomes small in the large N_c limit. In this way, the smallness of the expansion parameter m_ρ^2/Λ_χ^2 is justified in the large N_c QCD:

$$\frac{m_\rho^2}{\Lambda_\chi^2} = \frac{m_\rho^2}{(4\pi F_\pi)^2} \sim \frac{1}{N_c} \ \ll \ 1 \ . \tag{7}$$

One should note that this argument is true for any models including ρ, which is not enough for the existence of a systematic expansion.

For the existence of a systematic expansion, one needs to show that the contribution proportional to $1/m_\rho$ never appears at any loop order. This is actually guaranteed by the gauge invariance in the HLS theory, while there is no such argument in other models in my best knowledge[#2]. For example, when we include ρ as the matter field in the chiral Lagrangian, the form of the propagator of ρ is given by

$$\frac{1}{p^2 - m_\rho^2} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{m_\rho^2} \right] , \tag{8}$$

which coincides with the ρ propagator in the unitary gauge of the HLS (Weinberg's ρ meson [24]). The longitudinal part ($p_\mu p_\nu$-part) carries the factor of $1/m_\rho^2$ which may generate quantum corrections proportional to some powers of $1/m_\rho^2$. Appearance of a factor $1/m_\rho^2$ is a disaster in the loop calculations, particularly when the ρ mass is light. Namely, the derivative expansion discussed above breaks down. We note that the situation is similar in the "Massive Yang-Mills" approach and the "anti-symmetric tensor field method".

In the HLS, however, the gauge invariance prevent such a $1/m_\rho^2$ factor from appearing. This can be easily seen by the following ρ propagator in an R_ξ-like gauge fixing [8]:

$$\frac{1}{p^2 - m_\rho^2} \left[g_{\mu\nu} - (1-\alpha) \frac{p_\mu p_\nu}{p^2 - \alpha m_\rho^2} \right] , \tag{9}$$

where α is the gauge fixing parameter. The propagator in Eq. (9) is well defined in the limit of $m_\rho \to 0$ except for the unitary gauge ($\alpha = \infty$), while the propagator in Eq. (8) is ill-defined in such a limit. In addition, the gauge invariance guarantees that all the interactions never include a factor of $1/g^2 \propto 1/m_\rho^2$, while it may exist for the lack of the gauge invariance. Then all the loop corrections are well defined even in the limit of $m_\rho \to 0$. Thus the HLS gauge invariance is essential to performing the above derivative expansion. This makes the HLS most powerful among various methods for including the vector mesons based on the chiral symmetry.

One may think that the scalar mesons should be included, since several analysis [25] shows that they are lighter than the vector mesons in real-life QCD. For example, the analysis in Ref. [26] shows that the mass of sigma meson is about 560 MeV, which is definitely lighter than the ρ meson mass, $m_\rho = 770$ MeV. In the large N_c limit of QCD, however, it is natural to assume that the *light* sigma meson (flavor singlet scalar meson) does not exist in the following sense: There are two major pictures on the composition of the sigma meson, i.e., 2-quark picture and 4-quark picture [27, 28]. In the 2-quark picture, the sigma meson is made of one quark and one anti-quark and it is much lighter than the $a_0(980)$ meson due to the instanton effect [29]. Then, in the 2-quark picture, one can expect that the mass of the sigma meson becomes heavier and agree with the mass of $a_0(980)$ meson in the large N_c limit of QCD.

[#2] For recent attempts to include the effects of dynamical ρ in models other than the HLS, see, e.g., Ref. [23].

In the 4-quark picture, on the other hand, the sigma meson is made of two quarks and two anti-quarks, which does not exist in the large N_c limit of QCD [27, 30]. Furthermore, recently in Ref. [31], the analysis adopted in Refs. [32, 26] was extended for studying the π-π scattering in the real-life QCD to the one in the large N_c QCD, and it was shown that, for $N_c \geq 6$, the unitarity in the scalar channel of the π-π scattering is satisfied without scalar mesons up until the energy scale of $4\pi F_\pi$. This indicates that we do not need scalar mesons in the low-energy region in the large N_c QCD. In the real-life QCD the sigma meson is lighter than the ρ meson, but it is actually very broad. I expect that loop corrections from such broad resonances are very small, and that the chiral perturbation in the HLS is still possible, as far as we do not see the scalar channel.

Now that I explained that the smallness of the expansion parameter m_ρ^2/Λ_χ^2 is justified in the large N_c QCD, and that no appearance of quantum corrections proportional to some powers of $1/m_\rho^2$, I show the chiral order counting in the ChPT with HLS. As in the ordinary ChPT, the derivative and the external gauge fields are counted as $\mathcal{O}(p)$: $\partial_\mu \sim \mathcal{L}_\mu \sim \mathcal{R}_\mu \sim \mathcal{O}(p)$. In the HLS, ρ acquires its mass through the Higgs mechanism, which implies that the ρ mass m_ρ is proportional to the gauge coupling g. Then, the smallness of m_ρ/Λ_χ is achieved by the small gauge coupling. The expansion of the HLS is done by considering the expansion parameter m_ρ/Λ_χ to be equally small as p/Λ_χ. Thus, the gauge coupling is counted as order p [7, 9]:

$$g \sim \mathcal{O}(p). \tag{10}$$

This is the most important part of the ChPT with HLS. Using the above counting scheme, one can show that the loop expansion corresponds to the low-energy expansion and systematically calculate quantum corrections to several physical quantities based on the ChPT with HLS.

According to the entire list shown in Ref. [9], there are 35 counter terms at $\mathcal{O}(p^4)$ for general N_f. However, only three terms are relevant in the present analysis in which I consider two-point functions in the chiral limit:

$$\mathcal{L}_{(4)} = z_1 \mathrm{tr}[\hat{\mathcal{V}}_{\mu\nu}\hat{\mathcal{V}}^{\mu\nu}] + z_2 \mathrm{tr}[\hat{\mathcal{A}}_{\mu\nu}\hat{\mathcal{A}}^{\mu\nu}] + z_3 \mathrm{tr}[\hat{\mathcal{V}}_{\mu\nu}V^{\mu\nu}], \tag{11}$$

where

$$\hat{\mathcal{A}}_{\mu\nu} = \frac{1}{2}[\xi_R \mathcal{R}_{\mu\nu}\xi_R^\dagger - \xi_L \mathcal{L}_{\mu\nu}\xi_L^\dagger] \ , \tag{12}$$

$$\hat{\mathcal{V}}_{\mu\nu} = \frac{1}{2}[\xi_R \mathcal{R}_{\mu\nu}\xi_R^\dagger + \xi_L \mathcal{L}_{\mu\nu}\xi_L^\dagger] \ , \tag{13}$$

with $\mathcal{R}_{\mu\nu}$ and $\mathcal{L}_{\mu\nu}$ being the field strengths of $\mathcal{R}_\mu$ and $\mathcal{L}_\mu$.

3 Wilsonian Matching

The values of $\mathcal{O}(p^4)$ parameters as well as the leading order parameters F_π, a and g in the ChPT with HLS introduced in the previous section should be determined from the underlying

QCD. In Ref. [10], we proposed a way to determine some of the parameters by matching the HLS to QCD, which we called the Wilsonian matching. In this section, I will briefly review the Wilsonian matching.

Let me first explain the basic concept of the Wilsonian matching. In the high energy region, quarks and gluons are good degrees of freedom, and one can treat QCD in a perturbative way. In the low energy region, on the other hand, hadrons become good degrees of freedom instead of quarks and gluons. The main assumption of the Wilsonian matching is that there is some scale Λ at which both the perturbative QCD and the ChPT with HLS are applicable, and that one can switch the theory from QCD to the HLS at Λ. When there is such an overlapping energy region, the bare parameters of the HLS can be determined by matching the HLS with QCD at the matching scale Λ. In other words, the bare theory of the HLS can be obtained by integrating out the high energy modes, i.e., quarks and gluons, at Λ. Once the bare theory is determined, the quantum effect is included to relate the bare parameters with physical quantities such as the π decay constant and the ρ mass.

Since the procedure of the Wilsonian matching is a little different from the one adopted in several other EFTs, I show the essential difference starting with the basic concept of the EFT: The effective Lagrangian of the EFT, which has the most general form constructed from the chiral symmetry, gives the same generating functional as that obtained from QCD:

$$Z_{\mathrm{EFT}}[J,F] = \int \mathcal{D}U e^{iS_{\mathrm{eff}}[J,F]} \underset{\text{matching}}{\leftrightarrow} Z_{\mathrm{QCD}}[J] = \int \mathcal{D}q\mathcal{D}\bar{q}\mathcal{D}G e^{iS_{\mathrm{QCD}}[J]} \ , \tag{14}$$

where J is a set of external source fields. In the EFT side, U denotes a set of the relevant hadronic fields such as the pion fields, S_{eff} is the action expressed in terms of these hadrons, and F a set of parameters included in the EFT. In QCD side, q ($\bar{q}$) denotes (anti) quark field, G is the gluon field and S_{QCD} represents the action expressed in terms of the quarks and gluons.

In some matching schemes, the renormalized parameters of the EFT are determined by the matching. On the other hand, the matching in the Wilsonian sense is performed based on the following general idea: The bare Lagrangian of the EFT is defined at a suitable matching scale Λ and the generating functional derived from the bare Lagrangian leads to the same Green's function as that derived in QCD at Λ:

$$Z_{\mathrm{EFT}}[J,F]\big|_{E=\Lambda} = e^{iS_{\mathrm{eff}}[J,F_{\mathrm{bare}}]} \underset{\text{matching}}{\longleftrightarrow} Z_{\mathrm{QCD}}[J]\big|_{E=\Lambda} = \int \mathcal{D}q\mathcal{D}\bar{q}\mathcal{D}G e^{iS_{\mathrm{QCD}}[J]} \ , \tag{15}$$

where F_{bare} denotes a set of bare parameters. Through the above matching, the *bare* parameters of the EFT are determined. In other words, one obtains the bare Lagrangian of the EFT after integrating out the high energy modes, i.e., the quarks and gluons above Λ. Then the informations of the high energy modes are included in the parameters of the EFT.

In Refs. [10, 11], based on the above idea, the vector and axial-vector current correlators derived from the bare HLS theory are matched with those obtained by the operator product expansion (OPE) in QCD. It was shown that the physical predictions are in remarkable agreement with experiments. Furthermore, a recent analysis [12] shows that the Wilsonian

matching with the effect of current quark masses included reproduces the ratio f_k/f_π in remarkable agreement with experiment. I would like to stress that, for the above success of the Wilsonian matching, the effect of quadratic divergence plays an essential role: For obtaining the physical quantities starting from the *bare* theory, one, of course, has to include the effect of quadratic divergence into the RGEs in the Wilsonian sense.

Before going to the analysis of the chiral symmetry restoration based on the Wilsonian matching, let me discuss the validity of the systematic expansion in the Wilsonian matching procedure. One might think that the systematic expansion would break down near the matching scale, since the quadratic divergences from higher loops can in principle contribute to the $\mathcal{O}(p^2)$ terms. However, even when the quadratic divergences are explicitly included, the systematic expansion is still valid in the following sense: When one starts from the bare theory and calculates the quantum corrections including the quadratic divergence, the loop corrections are given in terms of the bare parameter $F_{\pi,\text{bare}}$ instead of the on-shell decay constant F_π. Then, the scale at which the theory breaks down should be

$$\Lambda_\chi \simeq 4\pi F_{\pi,\text{bare}} \ . \tag{16}$$

By using this chiral symmetry breaking scale, the quadratically divergent correction to the $\mathcal{O}(p^2)$ term at nth loop order takes the form of $[\Lambda^2/\Lambda_\chi^2]^n$. As for the ChPT with HLS explained in the previous section, the requirement $\Lambda < \Lambda_\chi$ is satisfied in the large N_c limit of QCD: In the large N_c limit of QCD, the quadratically divergent correction at nth loop order is suppressed by $[\Lambda^2/\Lambda_\chi^2]^n \sim [1/N_c]^n$. As a result, one can perform the systematic loop expansion with quadratic divergences included in the large N_c limit, and extrapolate the results to the real-life QCD. The quantitative analyses was done in Refs. [10, 11, 12], which show that the phenomenological analysis based on the Wilsonian matching together with the ChPT with HLS can be done in remarkable agreement with the experiments in much the same sense as the phenomenological analysis in the ordinary ChPT is successfully extended to the energy region higher than the pion mass scale, which is logically beyond the validity region of the ChPT.

4 Vector Manifestation of Chiral Symmetry

In this section, following Ref. [13, 11], I briefly review the difference between the vector manifestation (VM) and the conventional manifestation of chiral symmetry restoration based on the linear sigma model in terms of the chiral representation of the mesons by extending the analyses done in Refs. [33, 34] for two flavor QCD.

The VM was first proposed in Ref. [13] as a novel manifestation of Wigner realization of chiral symmetry where the vector meson ρ becomes massless at the chiral phase transition point. Accordingly, the (longitudinal) ρ becomes the chiral partner of the Nambu-Goldstone (NG) boson π. The VM is characterized by

$$\text{(VM)} \qquad f_\pi^2 \to 0 \ , \quad m_\rho^2 \to m_\pi^2 = 0 \ , \quad f_\rho^2/f_\pi^2 \to 1 \ , \tag{17}$$

where f_ρ is the decay constant of (longitudinal) ρ at ρ on-shell. This is completely different from the conventional picture based on the linear sigma model where the scalar meson S becomes massless degenerate with π as the chiral partner:

$$\text{(GL)} \qquad f_\pi^2 \to 0 \ , \quad m_S^2 \to m_\pi^2 = 0 \ . \tag{18}$$

In Ref. [11] this was called GL manifestation after the effective theory of Ginzburg–Landau or Gell-Mann–Levy.

I first consider the representations of the following zero helicity ($\lambda = 0$) states under $\mathrm{SU}(3)_{\mathrm{L}} \times \mathrm{SU}(3)_{\mathrm{R}}$; the π, the (longitudinal) ρ, the (longitudinal) axial-vector meson denoted by A_1 (a_1 meson and its flavor partners) and the scalar meson denoted by S. The π and the longitudinal A_1 are admixture of $(8\,,\,1) \oplus (1\,,\,8)$ and $(3\,,\,3^*) \oplus (3^*\,,\,3)$ since the symmetry is spontaneously broken [33, 34]:

$$\begin{aligned} |\pi\rangle &= |(3\,,\,3^*) \oplus (3^*\,,\,3)\rangle \sin\psi + |(8\,,\,1) \oplus (1\,,\,8)\rangle \cos\psi \ , \\ |A_1(\lambda=0)\rangle &= |(3\,,\,3^*) \oplus (3^*\,,\,3)\rangle \cos\psi - |(8\,,\,1) \oplus (1\,,\,8)\rangle \sin\psi \ , \end{aligned} \tag{19}$$

where the experimental value of the mixing angle ψ is given by approximately $\psi = \pi/4$ [33, 34]. On the other hand, the longitudinal ρ belongs to pure $(8\,,\,1) \oplus (1\,,\,8)$ and the scalar meson to pure $(3\,,\,3^*) \oplus (3^*\,,\,3)$:

$$\begin{aligned} |\rho(\lambda=0)\rangle &= |(8\,,\,1) \oplus (1\,,\,8)\rangle \ , \\ |S\rangle &= |(3\,,\,3^*) \oplus (3^*\,,\,3)\rangle \ . \end{aligned} \tag{20}$$

When the chiral symmetry is restored at the phase transition point, it is natural to expect that the chiral representations coincide with the mass eigenstates: The representation mixing is dissolved. From Eq. (19) one can easily see that there are two ways to express the representations in the Wigner phase of the chiral symmetry: The conventional GL manifestation corresponds to the limit $\psi \to \pi/2$ in which π is in the representation of pure $(3\,,\,3^*) \oplus (3^*\,,\,3)$ together with the scalar meson, both being the chiral partners:

$$\text{(GL)} \qquad \left\{ \begin{aligned} |\pi\rangle\,,|S\rangle &\to |(3\,,\,3^*) \oplus (3^*\,,\,3)\rangle \ , \\ |\rho(\lambda=0)\rangle\,,|A_1(\lambda=0)\rangle &\to |(8\,,\,1) \oplus (1\,,\,8)\rangle \ . \end{aligned} \right. \tag{21}$$

On the other hand, the VM corresponds to the limit $\psi \to 0$ in which the A_1 goes to a pure $(3\,,\,3^*) \oplus (3^*\,,\,3)$, now degenerate with the scalar meson S in the same representation, but not with ρ in $(8\,,\,1) \oplus (1\,,\,8)$:

$$\text{(VM)} \qquad \left\{ \begin{aligned} |\pi\rangle\,,|\rho(\lambda=0)\rangle &\to |(8\,,\,1) \oplus (1\,,\,8)\rangle \ , \\ |A_1(\lambda=0)\rangle\,,|s\rangle &\to |(3\,,\,3^*) \oplus (3^*\,,\,3)\rangle \ . \end{aligned} \right. \tag{22}$$

Namely, the degenerate massless π and (longitudinal) ρ at the phase transition point are the chiral partners in the representation of $(8\,,\,1) \oplus (1\,,\,8)$.

Next, I consider the helicity $\lambda = \pm 1$. note that the transverse ρ can belong to the representation different from the one for the longitudinal ρ ($\lambda = 0$) and thus can have the different chiral partners. According to the analysis in Ref. [33], the transverse components of ρ ($\lambda = \pm 1$) in the broken phase belong to almost pure $(3^*\,,\,3)$ ($\lambda = +1$) and $(3\,,\,3^*)$ ($\lambda = -1$) with tiny mixing with $(8\,,\,1) \oplus (1\,,\,8)$. Then, it is natural to consider in VM that they become pure $(3\,,\,3^*)$ and $(3^*\,,\,3)$ in the limit approaching the chiral restoration point [11]:

$$|\rho(\lambda = +1)\rangle \to |(3^*, 3)\rangle \ , \quad |\rho(\lambda = -1)\rangle \to |(3, 3^*)\rangle \ . \tag{23}$$

As a result, the chiral partners of the transverse components of ρ in the VM will be themselves. Near the critical point the longitudinal ρ becomes almost σ, namely the would-be NG boson σ almost becomes a true NG boson and hence a different particle than the transverse ρ.

5 Formulation of the Vector Manifestation in Hot Matter

In this section I briefly review how to formulate the vector manifestation (VM) in hot matter. I first show how to extend the Wilsonian matching to the version at non-zero temperature in order to incorporate the intrinsic thermal effect into the bare parameters of the HLS Lagrangian. Then, I briefly summarize how the VM is formulated in hot matter following Refs. [15, 35]. It should be noticed that the critical temperature of the chiral symmetry restoration is approached from the broken phase up to $T_c - \epsilon$, and that the following basic assumptions are adopted in the present analysis: (1) The relevant degrees of freedom until near $T_c - \epsilon$ are only π and ρ; (2) Other mesons such as a_1 and sigma mesons are still heavy at $T_c - \epsilon$; (3) Partial chiral symmetry restoration already occurs at $T_c - \epsilon$. Based on these assumptions, I will show that the VM necessarily occurs at the chiral symmetry restoration point.

Let me first explain how to extend the Wilsonian matching proposed at $T = 0$ [10] (see section 3) to the one at non-zero temperature following Ref. [35]. For this, it should be noticed that there is no longer Lorentz symmetry in hot matter, and the Lorentz non-scalar operators such as $\bar{q}\gamma_\mu D_\nu q$ may exist in the form of the current correlators derived by the OPE [36, 37]. This leads to, e.g., a difference between the temporal and spatial bare π decay constants. In the present analysis, however, I neglect the contributions from these operators since they give a small correction compared with the main term $1 + \frac{\alpha_s}{\pi}$. This implies that the Lorentz symmetry breaking effect in the bare π decay constant is small, $F^t_{\pi,\text{bare}} \simeq F^s_{\pi,\text{bare}}$ [38]. Thus it is a good approximation that I determine the π decay constant at non-zero temperature through the matching condition obtained at $T = 0$ in Ref. [10] with putting possible temperature dependences on the gluonic and quark condensates [15, 38]:

$$\frac{F_\pi^2(\Lambda; T)}{\Lambda^2} = \frac{1}{8\pi^2}\Big[1 + \frac{\alpha_s}{\pi} + \frac{2\pi^2}{3}\frac{\langle \frac{\alpha_s}{\pi} G_{\mu\nu}G^{\mu\nu}\rangle_T}{\Lambda^4} + \pi^3 \frac{1408}{27}\frac{\alpha_s \langle \bar{q}q\rangle_T^2}{\Lambda^6}\Big] \ . \tag{24}$$

Through this condition the temperature dependences of the quark and gluonic condensates determine the intrinsic temperature dependence of the bare parameter $F_\pi(\Lambda; T)$, which is then converted into those of the on-shell parameter $F_\pi(\mu = 0; T)$ through the Wilsonian RGE.

Now, let us consider the Wilsonian matching near the chiral symmetry restoration point assuming that the quark condensate approaches zero continuously for $T \to T_c$.[#3] First, note that the Wilsonian matching condition (24) provides

$$\frac{F_\pi^2(\Lambda; T_c)}{\Lambda^2} = \frac{1}{8\pi^2}\Big[1 + \frac{\alpha_s}{\pi} + \frac{2\pi^2}{3}\frac{\langle \frac{\alpha_s}{\pi} G_{\mu\nu}G^{\mu\nu}\rangle_{T_c}}{\Lambda^4}\Big] \neq 0 \ , \tag{25}$$

which implies that the matching with QCD dictates

$$F_\pi^2(\Lambda; T_c) \neq 0 \tag{26}$$

even at T_c where the on-shell π decay constant approaches zero by adding the quantum corrections through the RGE including the quadratic divergence [10] and the hadronic thermal correction [15, 35]. As was shown in Ref. [16] for the VM in dense matter, Lorentz non-invariant version of the VM conditions for the bare parameters are obtained by the requirement of the equality between the axial-vector and vector current correlators in the HLS, which should be valid also in hot matter [38]:

$$a^t_{\rm bare} \equiv \left(\frac{F^t_{\sigma,\rm bare}}{F^t_{\pi,\rm bare}}\right)^2 \underset{T\to T_c}{\longrightarrow} 1, \quad a^s_{\rm bare} \equiv \left(\frac{F^s_{\sigma,\rm bare}}{F^s_{\pi,\rm bare}}\right)^2 \underset{T\to T_c}{\longrightarrow} 1, \tag{27}$$

$$g_{T,\rm bare} \underset{T\to T_c}{\longrightarrow} 0, \quad g_{L,\rm bare} \underset{T\to T_c}{\longrightarrow} 0, \tag{28}$$

where $a^t_{\rm bare}$, $a^s_{\rm bare}$, $g_{T,\rm bare}$ and $g_{L,\rm bare}$ are the extensions of the parameters $a_{\rm bare}$ and $g_{\rm bare}$ in the bare Lagrangian with the Lorentz symmetry breaking effect included as in Appendix A of Ref. [16].

When we use the conditions for the parameters $a^{t,s}$ in Eq. (27) and the above result that the Lorentz symmetry violation between the bare π decay constants $F^{t,s}_{\pi,\rm bare}$ is small, we can easily show that the Lorentz symmetry breaking effect between the temporal and spatial bare σ decay constants is also small, $F^t_{\sigma,\rm bare} \simeq F^s_{\sigma,\rm bare}$ [38]. While we cannot determine the ratio $g_{L,\rm bare}/g_{T,\rm bare}$ through the Wilsonian matching since the transverse mode of ρ decouples near T_c. However, this implies that the transverse mode is irrelevant to the quantities studied in the present analysis. Therefore, I set $g_{L,\rm bare} = g_{T,\rm bare}$ for simplicity and use the Lorentz invariant Lagrangian at bare level. In the low temperature region, the intrinsic temperature dependences are negligible, so that one can use the Lorentz invariant Lagrangian at bare level as in the analysis by the ordinary chiral Lagrangian in Ref. [39].

As I discussed above, in a good approximation, one can start from the Lorentz invariant bare Lagrangian even in hot matter. In such a case the axial-vector and the vector current

[#3] Here and henceforth, I use just T_c which actually implies $T_c - \epsilon$.

correlators $G_A^{(\mathrm{HLS})}$ and $G_V^{(\mathrm{HLS})}$ are expressed by the same forms as those at zero temperature with the bare parameters having the intrinsic temperature dependences [15]:

$$\begin{aligned} G_A^{(\mathrm{HLS})}(Q^2) &= \frac{F_\pi^2(\Lambda;T)}{Q^2} - 2z_2(\Lambda;T), \\ G_V^{(\mathrm{HLS})}(Q^2) &= \frac{F_\sigma^2(\Lambda;T)[1-2g^2(\Lambda;T)z_3(\Lambda;T)]}{{M_\rho}^2(\Lambda;T)+Q^2} - 2z_1(\Lambda;T)\ . \end{aligned} \tag{29}$$

When the critical temperature is approached from below up to T_c, the axial-vector and vector current correlators derived in the OPE approach each other for any value of Q^2. Thus we require that these current correlators in the HLS become close to each other at T_c for any value of Q^2 around Λ^2. By taking account of the fact $F_\pi^2(\Lambda;T_c) \neq 0$ derived from the Wilsonian matching condition given in Eq. (25), the requirement $G_A^{(\mathrm{HLS})} \underset{T\to T_c}{\longrightarrow} G_V^{(\mathrm{HLS})}$ is satisfied only if the following conditions are met [15]:

$$\begin{aligned} & g(\Lambda;T) \underset{T\to T_c}{\longrightarrow} 0\ , \qquad a(\Lambda;T) \underset{T\to T_c}{\longrightarrow} 1\ , \\ & z_1(\Lambda;T) - z_2(\Lambda;T) \underset{T\to T_c}{\longrightarrow} 0\ . \end{aligned} \tag{30}$$

These conditions ("VM conditions in hot matter") for the bare parameters are converted into the conditions for the on-shell parameters through the Wilsonian RGEs. Since $g = 0$ and $a = 1$ are separately the fixed points of the RGEs for g and a [40], the on-shell parameters also satisfy $(g, a) = (0, 1)$, and thus the parametric ρ mass satisfies $M_\rho = 0$.

Now, let me include the hadronic thermal effects to obtain the ρ pole mass near T_c. As I explained above, the intrinsic temperature dependences imply that $M_\rho/T \to 0$ for $T \to T_c$, so that the ρ pole mass near the critical temperature is expressed as [15, 35]

$$m_\rho^2(T) = M_\rho^2 + g^2 N_f \, \frac{15-a^2}{144} T^2\ . \tag{31}$$

Since $a \to 1$ near T_c, the second term is positive. Then the ρ pole mass m_ρ is bigger than the parametric M_ρ due to the hadronic thermal corrections. Nevertheless, *the intrinsic temperature dependence determined by the Wilsonian matching requires that the ρ becomes massless at the critical temperature*:

$$m_\rho^2(T) \to 0 \quad \text{for } T \to T_c\ , \tag{32}$$

since the first term in Eq. (31) vanishes as $M_\rho \to 0$, and the second term also vanishes since $g \to 0$ for $T \to T_c$. This implies that *the vector manifestation (VM) actually occurs at the critical temperature* [15].

6 Summary

In this write-up, I first explained how the systematic chiral perturbation in the hidden local symmetry (HLS) is justified in the large N_c limit of QCD in section 2. Next in section 3, I

summarized the basic concept of the Wilsonian matching, through which some of the bare parameters of the HLS are determined by matching the HLS to the operator product expansion in QCD. In section 4 I showed the difference between the VM and the conventional manifestation of chiral symmetry restoration based on the linear sigma model in terms of the chiral representation of the mesons. Then, in section 5, I reviewed how to formulate the VM in hot matter.

There are several predictions of the VM in hot matter made so far. In Ref. [38], the vector and axial-vector susceptibilities were studied. It was shown that the equality between two susceptibilities are satisfied and that the VM predicts $\chi_A = \chi_V = \frac{2}{3}T_c^2$ for $N_f = 2$, which is in good agreement with the result obtained in the lattice simulation [41]. In Ref. [35], a prediction associated with the validity of vector dominance (VD) in hot matter was made: As a consequence of including the intrinsic effect, the VD is largely violated at the critical temperature. In addition to the above predictions, the pion velocity was studied including the effect of Lorentz symmetry breaking [42, 43]. It was shown that the pion velocity near T_c is close to the speed of light. Furthermore, in Ref. [44], starting with an HLS Lagrangian at the VM fixed point that incorporates the heavy-quark symmetry and matching the bare theory to QCD, we calculated the splitting of chiral doublers of D mesons proposed in Refs. [45, 46, 47], and showed that the splitting comes out to be $0.31 \pm 0.12\,\mathrm{GeV}$, which is in good agreement with the experiment [48, 49, 50]. Furthermore, the matching showed that the mass splitting is directly proportional to the light-quark condensate $\langle \bar{q}q \rangle$, which implies that the splitting vanishes at the chiral restoration point.

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